

AIR RESISTANCE w/ calc.

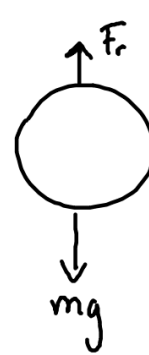
Terminal velocity is when a
goes to zero b/c $F_g = F_r$

$$\vec{F}_r = -b\vec{v}$$

$$= -b\vec{v}^2 \text{ "easy"}$$

 $v_0 = 0$ drop the ball
 $F_r = -kv$

find v_{terminal} .



1) FBD

2) $\Sigma F = ma$

$$mg - F_r = ma \xrightarrow{0} \text{ b/c } v_{\text{term.}}$$

$$mg = F_r$$

$$mg = kv$$

$$\frac{mg}{k} = v_{\text{term.}}$$



We need to find $v(t)$ for any t

From FBD we know $mg - kv = ma$

$$\frac{mg - kv}{m} = a$$

$$a = \frac{dv}{dt}$$

$$\frac{mg - kv}{m} = \frac{dv}{dt}$$

to solve for v , isolate
 v with dv

$$\frac{1}{m} = \frac{1}{mg - kv} \frac{dv}{dt}$$

move dt to other
side

$$\int \frac{1}{m} dt = \int \frac{1}{mg - kv} dv$$

integrate both sides

....

power rule does not work for

$$\int \frac{1}{x} dx \quad \text{b/c} \quad \int x^{-1} dx = \frac{x^0}{0}$$

$$\boxed{\int \frac{1}{x} dx = \ln|x| + C}$$

OH
No !!

$$\int \frac{1}{m} dt = \int \frac{1}{mg-bv} dv \quad \text{use u sub}$$

$\uparrow$ coefficient let $u = \overbrace{mg-bv}^{\text{constant}}$
 $\left(\frac{1}{m}\right)t + C = \int \frac{1}{u} dv \quad du = -b dv$

$$\frac{t}{m} + C = \int \frac{1}{u} \left(\frac{du}{-b}\right) \quad \frac{du}{-b} = dv$$

$\uparrow$ replace dv w/

$$\frac{t}{m} + C = -\frac{1}{b} \int \frac{1}{u} du$$

$$\frac{t}{m} + C_1 = -\frac{1}{b} \ln|u| + C_2$$

$$\frac{t}{m} + C = -\frac{1}{b} \ln|mg-bv| \quad \text{isolate } v$$

$$-\frac{bt}{m} + C = \ln|mg-bv|$$

$$e^{-\frac{bt}{m} + C} = e^{\ln|mg-bv|}$$

$$e^{-\frac{bt}{m}} \cdot e^C = mg-bv$$

$$C e^{-\frac{bt}{m}} = mg-bv$$

$$C e^{-\frac{bt}{m}} - mg = -bv$$

$$\frac{C e^{-\frac{bt}{m}} - mg}{-b} = v$$

$$\frac{mg}{b} - C e^{-\frac{bt}{m}} = v \quad \text{Solve for } C$$

plug in knowns

$$\frac{mg}{b} - C e^{-\frac{b(0)}{m}} = 0 \quad v_0, t_0$$

$v_0 = 0 @ t = 0$

$$-C e^0 = -\frac{mg}{b}$$

$$C = \frac{mg}{b} \quad \text{plug back in for } C$$

$$\frac{mg}{b} - \frac{mg}{b} e^{-\frac{bt}{m}} = v$$

$$\frac{mg}{b} (1 - e^{-\frac{bt}{m}}) = v \quad \text{done!!}$$